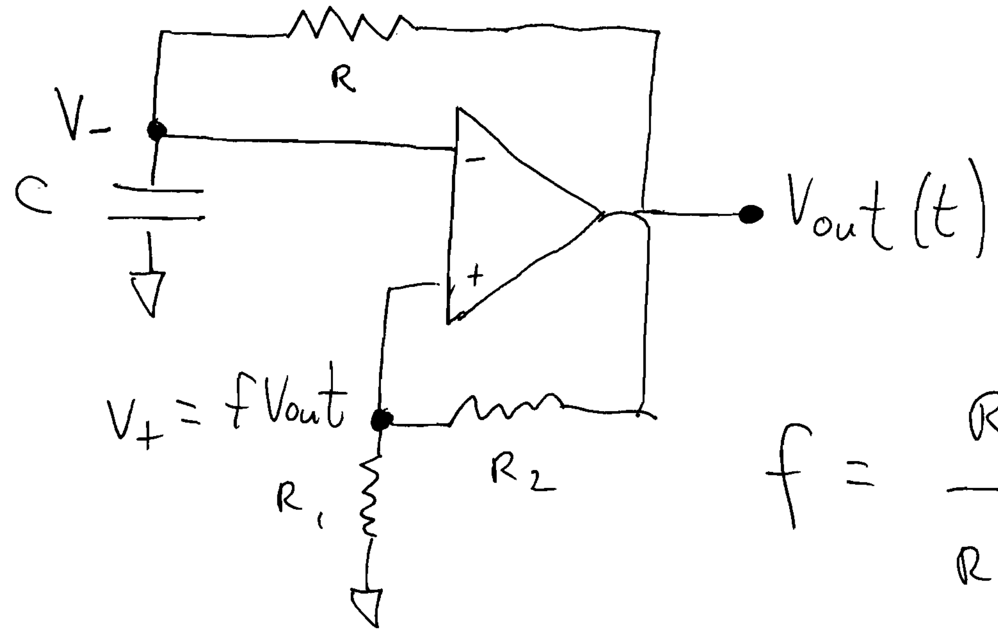
Op-Amp Oscillators :

Astable Multivibrator :



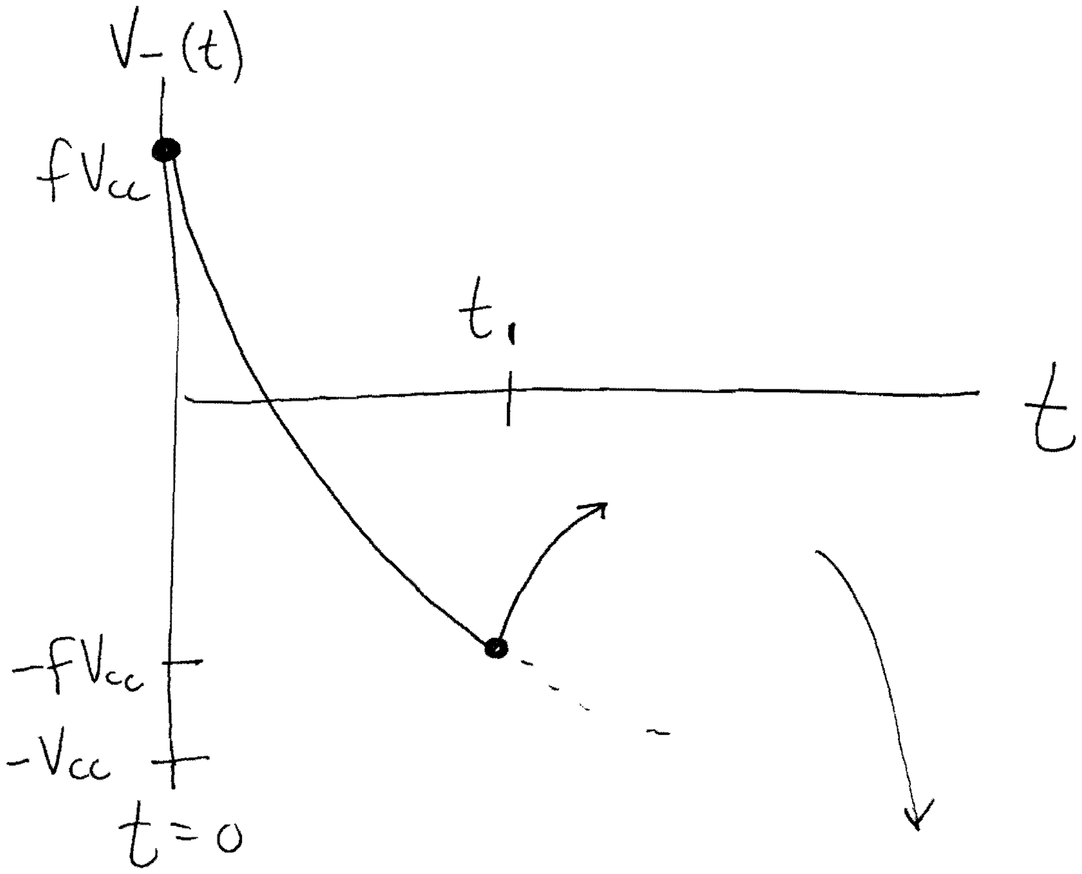
$$f = \frac{R_1}{R_1 + R_2}$$

Boundary Conditions :

Choose $t = 0$ to be when $V_{out}(t)$ has just switched from $+V_{cc}$ to $-V_{cc}$.

$\boxed{V_-(0) = f V_{cc} \text{ (Decaying)}}$ $V_+(0) = -f V_{cc}$

$\boxed{V_{out}(0) = -V_{cc}}$



Now write down $V_-(t)$ function.

$$\boxed{V_-(t) = V_{cc}(1+f)\,e^{-t/RC} - V_{cc}}$$

<u>check</u> $V_-(0) = fV_{cc}$ ✓

$V_-(\infty) = -V_{cc}$ (assyntotic limit)

<u>τ derivation :</u>

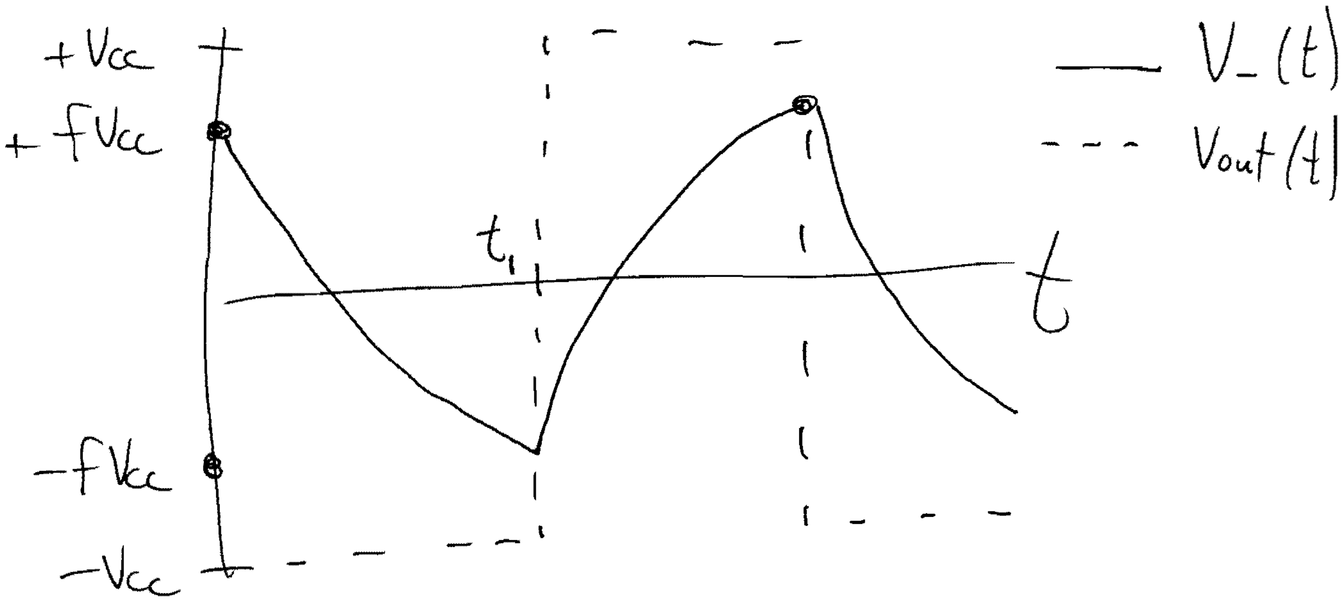
$$V_-(t_1) = -fV_{cc} = V_{cc}(1+f)\,e^{-t_1/RC} - V_{cc}$$

or $$-f = (1+f)\,e^{-t_1/RC} - 1 \qquad (2)$$

therefore, $\frac{1-f}{1+f} = e^{-t_1/RC}$

or equivalently $\boxed{t_1 = RC \ln\left(\frac{1+f}{1-f}\right)}$

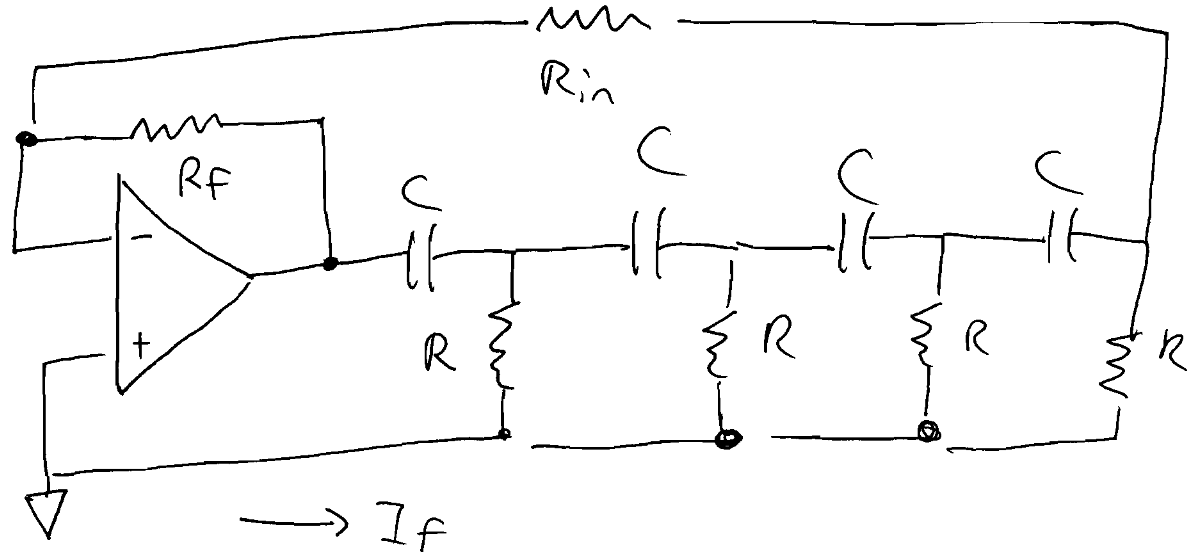
Recall the output waveforms...

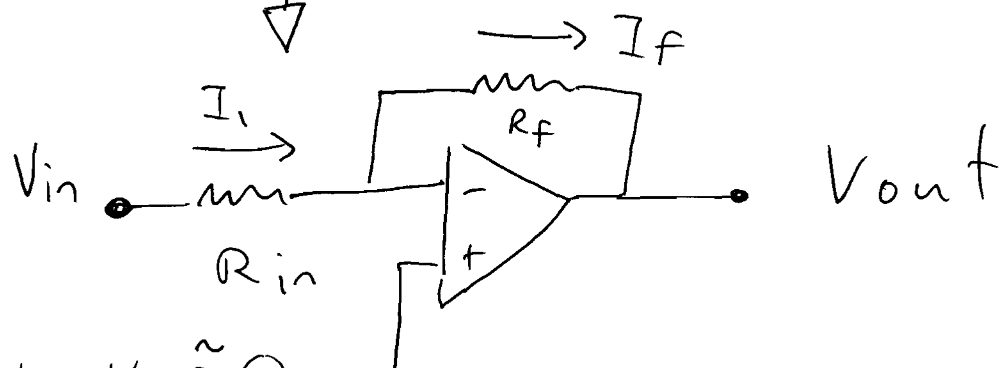


$2t_1 = T$ (Period of V_{out})

$$\Rightarrow \boxed{T = 2RC \ln\left(\frac{1+f}{1-f}\right)}$$

Phase shift oscillator:





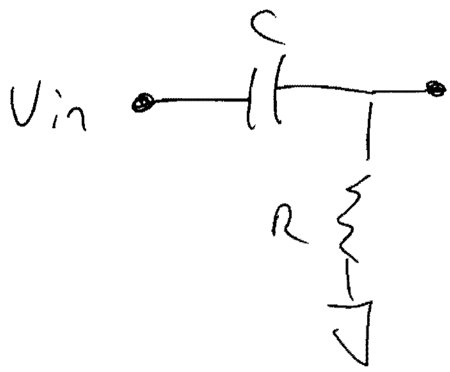
$V_+ = V_- \cong 0$

Assuming $\tilde{I}_B = 0$ $\quad I_1 = I_f$, or

$$\frac{V_{in} - V_-}{R_{in}} = \frac{V_- - V_{out}}{I_f} \Rightarrow \boxed{V_{out} = \frac{-R_f\, V_{in}}{R_{in}}}$$

It's <u>very</u> important to note that $G = G(\omega) \cong \frac{G_0}{1 + i\omega/\omega_0}$, so the output op-amp signal phase is <u>not</u> always π.

CR Filters: (Remember approximations when constructing A_{TOT})



$$V_{out} = \frac{R}{R + 1/i\omega C} V_{in}$$

$$\frac{V_{out}}{V_{in}} = A(\omega) = \frac{1}{1 + \frac{1}{i\omega RC}} = \frac{1}{1 - \frac{i}{\omega RC}}$$

$$\boxed{A(\omega) = \frac{e^{i\tan^{-1}\left(\frac{1}{\omega RC}\right)}}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}}}$$

Phase $\boxed{\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)}$

Oscillation occurs in these circuits when $\Delta\phi_{TOTAL}$ round trip is 2π. If $\phi_{op\text{-}amp} = \pi$, then the CR filters need to pick up the other π.

Ex: 4 CR filters make $\phi = \pi$

each must have $\phi_i = \frac{\pi}{4}$

$$\phi_{(\text{per stage})} = \tan^{-1}\left(\frac{1}{\omega RC}\right) = \frac{\pi}{4}$$

leads to $\boxed{\omega = \frac{1}{RC} \text{ produces } \phi_i = \frac{\pi}{4}}$

Ex: 2 CR filters

$$\tan^{-1}\left(\frac{1}{\omega RC}\right) = \frac{\pi}{2}$$

but $\tan^{-1}\left(\frac{\pi}{2}\right) = \infty$?

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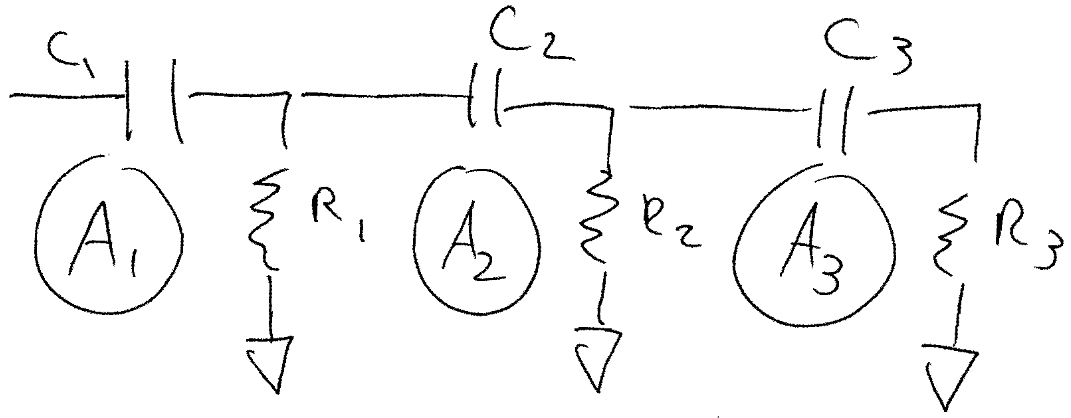
Exact Transmission Functions:

Due to the interaction between circuit elements multiplication of individual transmission functions is only an approximation for the exact transmission function.

Exact Method:



$$\boxed{\begin{gathered} A_1 = \frac{Z_1}{Z_1 + Z_{C_1}} = \frac{Z_1}{Z_1 + \frac{1}{i\omega C_1}} \\ Z_1 = R_1 \,||\, \left(Z_{C_2} + R_2 \,||\, \left(Z_{C_3} + R_3 \right) \right) \end{gathered}}$$

The previous Z_1 impedance is the circuit acting back on R_1.

$$\boxed{A_2 = \frac{Z_2}{Z_2 + Z_{C_2}} = \frac{Z_2}{Z_2 + \frac{1}{i\omega C_2}}}$$

$$\boxed{Z_2 = R_2 \parallel (Z_{C_3} + R_3)}$$

$$\boxed{A_3 = \frac{R_3}{R_3 + Z_{C_3}} = \frac{R_3}{R_3 + \frac{1}{i\omega C_3}}}$$

Useful to picture every object to the right "acting back" on previous stages to effect the transmission function.

$A_{TOT} = A_1 A_2 A_3$, where

$A_{1,2,3}$ include the modified

z_1, z_2, z_3 values which

encode all interaction.